New Budgeting and Risk Management

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Abstract

The paper develops contributions to the mathematical core of risk and uncertainty management in compliance with the principles of New Budgeting. A relevant cost function is presented and an event impact matrix with uncertain impacts is used to calculate the total risk budget. Uncertainty is represented in accordance with the two basic principles of probability and possibility, methods of calculations are presented, and comparisons are made by means of numerical examples. It is demonstrated that with identical numerical input the probability and possibility approaches produces distinctively different results.

Key words – New Budgeting, probability, possibility, risk management.

1. Introduction

Following a series of unacceptable budget overruns for large infrastructure projects, the Danish Government has announced that new principles for budgeting of public mega-projects should be instigated. Since the political document [1] was issued, the Ministry of Transportation is pioneering this New Budgeting initiative in a large project of developing and installing a very new signalling and security system to be implemented by the Danish railway authorities in years to come. Recent research confirms that new methods are required in order to avoid cost overruns, benefit shortfalls, and waste experienced in the majority of large projects, [2], [3], and [4]. This paper addresses the challenge of developing the mathematical core concepts in terms of alternative representations of risk and uncertainty in compliance with the New Budgeting principles in order to contribute to the development of new risk management methods and capabilities.

Alternative approaches to modelling of risk and uncertainty can be divided in two distinctive different categories, namely methods based on probabilities and possibilities, [5]. The classical approach is based on probability that can be interpreted as relative frequencies related to repetition of identical experiments or, more generally, uncertain knowledge or belief of a statistical nature. Criticism has been raised towards probability theory as being a too normative framework to take all the aspects of uncertain judgement into account, [6]. On the contrary, possibility relates to the degree of feasibility and ease of attainment or imprecise knowledge. Particularly, this paper proposes either approach to be used in connection with New Budgeting, depending on the type of risk and uncertainty at hand in the specific project.

Conventionally, risk and uncertainty is connected with the notion of something unwanted, something that should be avoided, like increased cost or deterioration of benefits. In this paper, we will consider risk and uncertainty also as an opportunity of improving things. Thus, a risk event is understood as a discrete occurrence or state of affairs that will influence the project for better or worse.

The paper is organised as follows: Section 2 gives a summary of the main points of New Budgeting with special relevance for risk and uncertainty management. In Section 3 a cost model compatible with the requirements in New Budgeting is developed. Section 4 presents an overview of conventional stochastic modelling. Section 5 introduces a possibility based representation and processing of uncertainty by intervals as the simplest type of a fuzzy number, which is extensively treated in Section 6. In Section 7 numerical examples are produced and compared. Finally, Section 8 contains a discussion and concluding remarks.

2. The principles of New Budgeting

Following the initiative [1], efforts has been made to put practical advices and guidelines concerning New Budgeting into existence, [7]. Some of the main principles of New Budgeting are:

- "Best realistic budget based on available knowledge." Unit prices and volumes are estimated based on experience from comparable projects. Cost of project management is budgeted specifically.
- Budget control is done by standardised budgets and logging of follow-up results. At the end of all project phases, a budget version is logged. Make explanation available on any difference between actual and earlier budgets.
- Risk and uncertainty management is conducted during entire project. IT-supported database of all risks and uncertainties is maintained. Continuous reporting on development of risks and uncertainties is done.

Focusing on economy, risk, and uncertainty, the main phases in New Budgeting are:

1. Analysis of alternatives, rough economic estimates. Creation of risk database containing immediately identifiable risks.
2. Creation of basis for decision. Detailed analysis of technical, economic, and organisational areas as well as call for tenders. Focus on the largest risk and uncertainty areas relating to the total budget and time schedule. Decision on further analysis of specific alternatives.

3. Detailed design planning.
4. Call for tenders.
5. Execution.

The rough budget estimate in Phase 1 is based on estimates of unit prices, volumes, and particular risks. An additional experience based supplementary budget of one third of 30% of the rough budget is allocated in order to arrive at the *anchor budget*, whereas the remaining two thirds are paid to a central reserve fund to cover the aggregated risk of the total portfolio of ongoing projects.

Specifically, the reporting from Phase 1 includes a preliminary assessment of risk and uncertainty. Further, a survey of the risk database including assessments of likelihoods of risk events and impacts on physical conditions and unit costs will be reported. Central to Phase 2 is the risk database that should be designed to contain all identified risks and uncertainties including likelihoods of occurrence and budgetary impacts, risk mitigation initiatives, and follow up actions. It is not acceptable to make use of the theoretical risk value determined by likelihood multiplied by impact.

3. A cost model for Risk Management

The anchor budget

For the purpose of this paper, we will focus on the project life cycle from the anchor budget onwards. In order to keep track of the way cost is developing during the project period under consideration we calculate the anchor budget at the time $t = 0$. Any subsequent modification to the anchor budget is marked by the time t of modification, so that comparison with the anchor budget, modifications previously made, and/or realised cost can be established at any time wanted.

Let the entire project be described by N activities. For the cost of the i'th activity of the project an estimated volume q_i at the estimated unit price p_i is needed resulting in the cost at the time $t = 0$

$$C_i = p_i \cdot q_i. \tag{1}$$

With a total of N activities this gives the total cost function C ($t = 0$) of the anchor budget

$$C = p_1 \cdot q_1 + p_2 \cdot q_2 + \ldots + p_i \cdot q_i + \ldots + p_N \cdot q_N. \tag{2}$$

Notice that some of the quantities may be equal to zero. This is because that subsequent risk analyses may create new types of activity costs compared to those relevant for the anchor budget. For the sake of simplicity we let N denote the total number of activity costs relevant for the completed project. This means that the value of N may not be formally known at the time $t = 0$ when the anchor budget is made with a number of activity costs less then or equal to N.

The risk budget

An analysis at the time t of possible risk events during the project results in a number M of possible risks that will influence the N activities more or less according to a thorough analysis. In general, any risk event may influence all of the activities concerning both price and quantity. Considering the j'th risk event it will influence the i'th price by the amount Δp_{ij} and the i'th quantity by the amount Δq_{ij}. The deltas may be positive or negative, representing an increase in unit price and quantity or a decrease, respectively.

Considering all M risk events we can write for the modified estimation of the i'th unit price p_i^t

$$p_i^t = p_i + \Delta p_{i1} + \Delta p_{i2} + \ldots + \Delta p_{ij} + \ldots + \Delta p_{iM}. \quad (3)$$

(For simplicity of notation, the symbol t is omitted for the deltas). Correspondingly, we have for the modified estimation of the i'th quantity q_i^t

$$q_i^t = q_i + \Delta q_{i1} + \Delta q_{i2} + \ldots + \Delta q_{ij} + \ldots + \Delta q_{iM}. \quad (4)$$

For the i'th activity, we then get the modified estimated cost C_i^t at time t

$$\begin{aligned}
C_i^t &= p_i^t \cdot q_i^t \\
&= (p_i + \Delta p_{i1} + \Delta p_{i2} + \ldots + \Delta p_{ij} + \ldots + \Delta p_{iM}) \cdot (q_i + \Delta q_{i1} + \Delta q_{i2} + \ldots + \Delta q_{ij} + \ldots + \Delta q_{iM}) \\
&= C_i + p_i \cdot (\Delta q_{i1} + \Delta q_{i2} + \ldots + \Delta q_{ij} + \ldots + \Delta q_{iM}) + (\Delta p_{i1} + \Delta p_{i2} + \ldots + \Delta p_{ij} + \ldots + \Delta p_{iM}) \cdot q_i \\
&\quad + \Delta p_{i1} \cdot (\Delta q_{i1} + \Delta q_{i2} + \ldots + \Delta q_{ij} + \ldots + \Delta q_{iM}) \\
&\quad + \Delta p_{i2} \cdot (\Delta q_{i1} + \Delta q_{i2} + \ldots + \Delta q_{ij} + \ldots + \Delta q_{iM}) \\
&\quad + \ldots \\
&\quad + \Delta p_{ij} \cdot (\Delta q_{i1} + \Delta q_{i2} + \ldots + \Delta q_{ij} + \ldots + \Delta q_{iM}) \\
&\quad + \ldots \\
&\quad + \Delta p_{iM} \cdot (\Delta q_{i1} + \Delta q_{i2} + \ldots + \Delta q_{ij} + \ldots + \Delta q_{iM}).
\end{aligned} \quad (5)$$

In case the deltas are small compared to unit price p_i and quantity q_i, products of deltas can be neglected, and the modified estimated cost C_i^t of the i'th cost item at time t can be approximated by

$$C_i^t \cong C_i + p_i \cdot (\Delta q_{i1} + \Delta q_{i2} + \ldots + \Delta q_{ij} + \ldots + \Delta q_{iM}) + (\Delta p_{i1} + \Delta p_{i2} + \ldots + \Delta p_{ij} + \ldots + \Delta p_{iM}) \cdot q_i. \quad (6)$$

Notice that for activities not present in the anchor budget but generated by subsequent risk analyses, products of the deltas can generally not be ignored, because they represent large changes relative to cost originally estimated to be equal to zero.

By nature, risk analyses may turn out differently at different times. This is due to the fact, that attempts may have been made to reduce the risk and new knowledge has been acquired (may be at an additional cost). In addition, change of design specifications during the project is a frequent cause of new activities and derived costs. Naturally, if an activity is completed, the realised unit price and quantity is introduced in the calculations since they are no more subject to risk or change.

Uncertainty of risk events occurring

Obviously, by nature a risk event is characterised by the fact that it may or may not occur. Other approaches assume likelihoods of risk events occurring, thus creating the challenge of estimating such likelihoods, [8], [9]. Certainly, the risk manager is left with the challenge of determining which events to include in the risk analysis and which not. Should many events with large likelihoods of occurring but small impacts be given the same priority as one event with a small likelihood of occurring but a large impact? In this paper, we propose to take the approach that all risk events judged relevant in terms of possible impact are included in the calculations. This means that risk events are included if the estimated impact is possibly large enough to significantly influence the resulting cost, no matter if the likelihood of occurrence is small or large. Any estimation of likelihoods of occurrence may be carried in a separate risk database and used during the risk mitigation process.

Uncertainty of impacts of risks events

Whereas we include any risk event in the analysis according to relevance criteria, there is still the problem of assessing and representing the size of the impact of the risk event, which is not normally done in any of the known standard procedures. In this paper, we take the approach that such impacts can only be estimated by uncertainty depending on the body of knowledge and experience available to the decision maker. Any such estimation will have to be expressed in terms of changes of unit price and quantity for the already known cost items or, alternatively, unit price and quantity for new cost items to be introduced in the risk budget. Actually, even the anchor budget may be expressed in terms of uncertain unit prices and quantities if, for any reason, uncertainty is prevailing at the time $t = 0$, when the anchor budget is set up. Consequently, in the remainder of the paper, any unit price and quantity, as well as any changes to them, may be represented by an uncertain number. Since there are a number of ways to represent uncertainty and carry out the related computations, this will be further elaborated in the following sections.

4. Conventional stochastic modelling

The traditional approach to representation of uncertainty in economic theory is that of probabilities. An uncertain economic parameter is represented by a precise probability distribution reflecting the variability of the parameter according to the objective knowledge or, alternatively, the subjective belief of the decision-making agent.

A conventional stochastic variable

$$X = \{\mu; \sigma\} \tag{7}$$

is characterized by the expected value $E(X) = \mu$ and the variance $VAR(X) = \sigma^2$, where σ is as the standard deviation.

Let X_1 and X_2 be independent stochastic variables with expected values $E(X_1) = \mu_1$ and $E(X_2) = \mu_2$ and variances $VAR(X_1) = \sigma_1^2$ and $VAR(X_2) = \sigma_2^2$. In the general case where the actual economic problem under consideration can be modelled by a function Y of n independent stochastic variables

$$Y = Y(X_1, X_2,\ldots, X_n) \tag{8}$$

we can approximate Y by means of a Taylor series (ignoring second and higher order terms)

$$Y \cong Y(\mu_1,\ldots, \mu_n) + \partial Y/\partial X_1\cdot(X_1-\mu_1) + \partial Y/\partial X_2\cdot(X_2-\mu_2) + \ldots + \partial Y/\partial X_n\cdot(X_n-\mu_n) \tag{9}$$

where $\partial Y/\partial X_i$ is the partial derivative of Y with respect to X_i calculated at $(\mu_1,\ldots, \mu_n)$. The expected value μ is given by

$$E(Y) = \mu = Y(\mu_1, \ldots, \mu_n) \tag{10}$$

whereas the variance σ^2 is approximated by

$$VAR(Y) = \sigma^2 \cong (\partial Y/\partial X_1)^2 \cdot \sigma_1^2 + \ldots + (\partial Y/\partial X_n)^2 \cdot \sigma_n^2. \tag{11}$$

Obviously, in order to evaluate the results of a particular model in terms of μ and σ an explicit formula (8) must be constructed and partial derivatives with respect to all variables (9) must be calculated. The procedure is quite simple in it self and does not require specific knowledge of the probability distributions of the variables involved besides the expected values and standard deviations.

A remark on dependent variables: According to [10, p. 111] most people intuitively agree that an increased degree of detailing of the variables X in (8) will reduce the resulting uncertainty of the function Y, however only to a certain limit. The subdivision should be done according to the "successive principle" in an "intelligent" way so that the resulting sub-variables are statistically independent and the assumed original dependence is accounted for by introducing correction factors. It is further claimed that practical evidence has been established in support of this although no mathematical proof has been published.

Monte Carlo simulation has become a standard technique in obtaining solutions to complex uncertainty models. The basic procedure is to represent uncertain variables by specific probability distributions and to perform a large number of model calculations based upon randomly generated values of the parameters. Finally, upon a large number of calculations, standard statistical methods are applied to characterize and present the uncertainty of the output parameters. This paper makes use of a commercially available program as an add-in module for MS-Excel, namely @Risk 5.0 [11], for carrying out Monte Carlo simulations.

5. Representation and processing of uncertainty by intervals

Following [12], [13], and [14] we define a real interval number as an ordered pair [a; b] of real numbers with $a \leq b$. It may also be defined as an ordinary set of real numbers x such that $a \leq x \leq b$, or

$$[a; b] = \{x \mid a \leq x \leq b\}. \tag{12}$$

If the basic arithmetic operations addition, subtraction, multiplication, and division are denoted by the symbol #, we can define operations on two intervals $\mathbf{I}_1 = [a_1; b_1]$ and $\mathbf{I}_2 = [a_2; b_2]$ based on the set-theoretic formulation:

$$\mathbf{I}_1 \# \mathbf{I}_2 = \{x \# y \mid a_1 \leq x \leq b_1, a_2 \leq y \leq b_2\}. \tag{13}$$

For basic operations on the intervals $\mathbf{I}_1$ and $\mathbf{I}_2$ we get the resulting interval $\mathbf{I} = [a; b]$ by the formulas

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = [a_1+a_2; b_1+b_2],$$

$$\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2 = [a_1-b_2; b_1-a_2],$$

$$\mathbf{I} = \mathbf{I}_1 \cdot \mathbf{I}_2 = [\min(a_1 \cdot a_2, a_1 \cdot b_2, b_1 \cdot a_2, b_1 \cdot b_2); \max(a_1 \cdot a_2, a_1 \cdot b_2, b_1 \cdot a_2, b_1 \cdot b_2)];$$

$$\mathbf{I} = \mathbf{I}_1 / \mathbf{I}_2 = [\min(a_1/a_2, a_1/b_2, b_1/a_2, b_1/b_2); \max(a_1/a_2, a_1/b_2, b_1/a_2, b_1/b_2)], 0 \notin [a_2; b_2]. \tag{14}$$

It can be shown that the four basic interval operations are inclusion monotonic, commutative, and associative. However, the distributive rule is not valid in general. Instead, the so-called sub-distributivity holds

$$\mathbf{I}_1 \cdot (\mathbf{I}_2 + \mathbf{I}_3) \subseteq \mathbf{I}_1 \cdot \mathbf{I}_2 + \mathbf{I}_1 \cdot \mathbf{I}_3. \tag{15}$$

From a rational real valued function F of n real valued variables

$$F = F(x_1, x_2, \ldots, x_n) \tag{16}$$

we can create the interval extension function as an interval function $\mathbf{F}^I$ of n intervals

$$\mathbf{F}^I = \mathbf{F}^I(\mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I}_n) \tag{17}$$

simply by replacing the real operators by interval operators and the real variables by intervals.

A rational function can be formulated in many ways whereas the same reformulations cannot be done for interval expressions due to the invalidity of the distributive rule. This implies that different formulations of a rational function will lead to different interval extension functions and thus to different interval results. In the case of F being a monotonic function within the entire range of the input variables the minimum and maximum of $\mathbf{F}^I$ as an interval can simply be found among the function values F at the extreme points of the variables. In the general case of F being non-monotonic or variables appearing more then once, the calculation of $\mathbf{F}^I$ as an interval is non-trivial. This is demonstrated in the following example.

Example: Based on the function $F = x \cdot (1 - x)$ the interval function $\mathbf{F}^I = \mathbf{I} \cdot (1 - \mathbf{I})$, $\mathbf{I} = [0; 1]$ is calculated. Straightforward application of the formulas from (11) gives the result $\mathbf{F}^I = [0; 1]$ whereas the correct result is [0; 0,25].

In this paper, the term "correct" is used to indicate the narrowest possible interval that can be calculated for an uncertain variable. Generally, to obtain this, iterative global optimization methods have to be used, see e.g. [15] and [16]. In order to obtain correct results (as in the above example) to an accuracy specified by the user, interval calculations in this paper are carried out using the Interval Solver 2000 program, [17] and [18], as an add-in module to MS-Excel 2000. An overall absolute and relative precision of 10^{-6} has been applied.

Correct calculation of interval functions allows for strong statements about the imprecision involved. *Firstly*, you can say that provided all uncertain input variables stay within their minimum and maximum values, the uncertain output function will stay within its minimum and maximum values. *Secondly*, the uncertain output function will not attain any value that is not a function value of some combination of the uncertain input values (within their minimum and maximum values). Application of intervals to investment analysis was done in [19] and [20].

6. Representation and processing of uncertainty by fuzzy numbers

A fuzzy set A in X where X is a space of points (objects) with a generic element of X denoted by x, i.e. X = {x}, is characterized by a membership function $f_A(x)$ which associates with each point in X a real number in the interval [0; 1]. The value of the membership function $f_A(x)$ at x represents the "grade of membership" of x in A. Thus the closer the value of $f_A(x)$ to unity, the higher the grade of membership of x in A. Note that when A is an ordinary set, i.e. non-fuzzy, the membership function can take only two values 0 and 1. In other words, a fuzzy set is a set of ordered pairs $(x, f_A(x))$

$$A = \{(x, f_A(x)) \mid x \in X\}. \qquad (18)$$

It is also useful to define the ordinary (non-fuzzy) set A_α as the α-cut of A:

$$A_\alpha = \{x \in X \mid f_A(x) \geq \alpha, 0 \leq \alpha \leq 1\}. \quad (19)$$

In this paper, we are mainly interested in the concept of fuzzy numbers as a means of representing uncertain or fuzzy information, [21] and [22]. In addition to the simplest fuzzy number, namely the interval, we also make use of the triangular fuzzy number, see [23], [a; c; b] where $a \leq c \leq b$, that can be defined by its membership function:

$$\begin{aligned} f(x) &= (x-a)/(c-a), \quad a \leq x \leq c, \\ &= (b-x)/(b-c), \quad c \leq x \leq b, \\ &= 0, \quad \text{otherwise.} \end{aligned} \quad (20)$$

Mathematical operations on triangular fuzzy numbers can be facilitated by introducing the left $L(\alpha)$ and right $R(\alpha)$ representation of a fuzzy triangular number $\mathbf{F}^T$, refer to the α-cut (19):

$$\mathbf{F}^T = [L(\alpha); R(\alpha)], \text{ where } L(\alpha) = a + (c-a)\alpha \text{ and } R(\alpha) = b + (c-b)\alpha, \alpha \in [0, 1]. \quad (21)$$

Observe that in this notation a fuzzy number is written as an interval with upper and lower bounds depending on α. This means that addition, subtraction, multiplication, and division can be carried out by using interval methods for all values of α. Likewise, for any function, the resulting functional values can be calculated and represented by L and R functions using interval methods for all values of α.

Example: Based on the real valued function $F = x \cdot (1 - x)$ calculate the corresponding fuzzy function with triangular argument [0; 0,5; 1]. Correct results have been calculated with global optimization and are shown in the table below. It can be seen from the table that F has a maximum of 0,25 at x = 0,5, which

α	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
$L(\alpha)$	0,000	0,047	0,090	0,127	0,160	0,188	0,210	0,227	0,240	0,248	0,250
$R(\alpha)$	0,250	0,250	0,250	0,250	0,250	0,250	0,250	0,250	0,250	0,250	0,250

corresponds to $\alpha = 1$. To obtain simpler representations and reduce the number of calculations triple representations of fuzzy variables corresponding to α-cuts 0 and 1 may be used. In the triangular case, the correct result is [0; 0,25; 0,25].

For a numerical comparison of alternative approaches to modelling of economic uncertainty, [24] should be consulted. In [25] a unified approach, combining probability and possibility, is presented. Specific calculatory difficulties were treated in [26].

7. Some computational results

A convenient calculation format

The model from Section 3 can be set up in a convenient calculation format that will allow for easy communication with professionals from economy as well as mathematics, see Table 1. Note that the parameters in Table 1 can be represented by conventional figures (no uncertainty is present) as well as probability distributions, intervals, or fuzzy numbers (uncertainty is present). Depending on the representation chosen the processing of the input data will have to be done differently as previously described in Sections 4, 5, and 6.

A numerical example

For the purpose of demonstrating the principles proposed in this paper, a simple example with five activities and three risk events is introduced. In Table 2 the numerical calculations are shown in the case where no uncertainty is present as far as the impacts are concerned. Note, that Risk Event No. 3 creates a new activity not included in the original Anchor Budget, but it also influences Activity No. 3. It turns out that the impacts result in a total cost increase of 11,2% at the time t, relative to the Anchor Budget.

Triangular representation of uncertainty

When the impacts are known only with uncertainty we choose to represent them by three real numbers a, c, and b, where $a \leq c \leq b$. This triangular representation of uncertainty can be interpreted in two different ways corresponding to possibility and probability. In the first case, we have a triple estimate of a triangular fuzzy number [a; c; b] with the membership function (20). The second case is a triangular probability distribution of the form (7), where $\mu = (a + b + c)/3$ and $\sigma^2 = (a^2 + b^2 + c^2 - ab - ac - bc)/18$ and the maximum probability h is attained at c. By normalisation $h = 2/(b-a)$. Table 3 contains the triangular values of the impacts used in the example. The dual interpretation using numerically identical impact matrices allows for a comparison of the probability and the possibility approach which will be discussed later.

Representing uncertainty by triple estimates

The Risk Budget is found in Table 4 under the assumption that the impacts in Table 3 are triple estimates. It is seen that the total cost is calculated to be the triple estimate [931.288; 945.000; 982.000]. This means that the cost is not lower than 931.288 and not higher than 982.000. The most possible cost is 945.000 (compare with Table 2). The cost increase produced by the risk analysis at time t is the triple estimate [9,6%; 11,2%; 15,5%] relative to the Anchor Budget. This means that the cost increase is not lower than 9,6% and not higher than 15,5%. The most probable cost increase is 11,2% (compare with Table 2).

Representing uncertainty by triangular probability distributions

Here the assumption is that the impacts in Table 3 are triangular probability distributions and that there are no correlations between the input parameters. By using Monte Carlo simulation, the Risk Budget can be calculated and the resulting probability distribution of the total cost is shown in Fig. 1. Written on the form (7) the cost is

$$C^t = \{952.463;\ 4.308\}. \tag{22}$$

The standard deviation is remarkably small. By an increasing number of independent parameters, the standard deviation usually gets smaller. From the cumulative distribution corresponding to Fig. 1, it can be seen that with probability 0,1 the total cost is less than ~947.000 and with probability 0,9 less than ~958.000.

A similar simulation has been run with the assumption of fully correlated input parameters and the result is shown in Fig. 2. The resulting cost can be written

$$C^t = \{925.538;\ 10.675\}. \tag{23}$$

The standard deviation has more than doubled compared to independent parameters. With probability 0,1 the total cost is less than ~939.000 and with probability 0,9 less than ~968.000.

8. Discussion and conclusion

Attempts have been made to contribute to the mathematical core of risk and uncertainty management relevant to the principles of New Budgeting as proposed by the Danish Ministry of Transportation. It has been of special importance to develop a convenient way of setting up the calculation procedure in order to match the requirements of New Budgeting in terms of ease of communication, traceability of changes throughout the entire project life cycle, and representation of risk and uncertainty. Specifically, the approach has been taken to include all risk events with major impacts in the calculations no matter if the likelihood of occurrence is judged small or large. On the other hand, the methodology developed allow for representation of uncertainty of impacts based on either probabilities or possibilities. The former approach appear to specifically well suited in case of uncertainty of a statistical or variational nature whereas the latter is more suited to represent lack of knowledge or imprecision. As made clear in the paper the two alternative approaches require methods of propagation that are quite different by nature and ways of handle the computations have been presented. By way of using numerically identical triangular uncertain input variables, the distinct differences and similarities in outcomes are demonstrated. Obviously, the uncertainty of the total cost is much larger in the possibility case than in the probability case with independent input parameters. For example, by consulting Fig. 1 it is readily seen that the probability of the total cost coming out larger than 965.000 is practically equal to zero. From Table 4 it is definitely quite possible that this could happen. Thus, the probability approach may lead to underestimation of uncertainty, especially where the assumption of statistically independent variables is made. For an inspiring account of the fallacy of small probabilities being ignored with disastrous consequences, see [27]. Application of the possibility approach instead of probability can be a way of coping. On the other hand, when using fully correlated input parameters, it is seen from Fig. 2 that the triangular form of the input parameters is reproduced in the total cost. The triple estimate obtained by the possibility approach [931.288; 945.000; 982.000] (see Table 4) is closely approximated by the Monte Carlo result [931.281; 944.656; 981.898] (see Fig. 2). The results presented in this paper will be further developed in ongoing research.

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Table 1. Practical set-up for calculations, N = 5, M = 3.

	Anchor Budget at time t = 0			Event Impact Matrix, analysed at time t						Risk Budget at time t				
				1		2		3						
Activity	**p**	**q**	**C**	**Δp**	**Δq**	**Δp**	**Δq**	**Δp**	**Δq**	**p**	**q**	**C^t**	**ΔCost**	**ΔCost %**
1	p_1	q_1	$C_1=p_1 \cdot q_1$	Δp_{11}	Δq_{11}	Δp_{12}	Δq_{12}	Δp_{13}	Δq_{13}	$p_1^t=p_1+\sum\Delta p_{1j}$	$q_1^t=q_1+\sum\Delta q_{1j}$	$C_1^t=p_1^t \cdot q_1^t$	$C_1^t - C_1$	$C_1^t/C_1 - 1$
2	p_2	q_2	$C_2=p_2 \cdot q_2$	Δp_{21}	Δq_{21}	Δp_{22}	Δq_{22}	Δp_{23}	Δq_{23}	$p_2^t=p_2+\sum\Delta p_{2j}$	$q_2^t=q_2+\sum\Delta q_{2j}$	$C_2^t=p_2^t \cdot q_2^t$	$C_2^t - C_2$	$C_2^t/C_2 - 1$
3	p_3	q_3	$C_3=p_3 \cdot q_3$	Δp_{31}	Δq_{31}	Δp_{32}	Δq_{32}	Δp_{33}	Δq_{33}	$p_3^t=p_3+\sum\Delta p_{3j}$	$q_3^t=q_3+\sum\Delta q_{3j}$	$C_3^t=p_3^t \cdot q_3^t$	$C_3^t - C_3$	$C_3^t/C_3 - 1$
4	p_4	q_4	$C_4=p_4 \cdot q_4$	Δp_{41}	Δq_{41}	Δp_{42}	Δq_{42}	Δp_{43}	Δq_{43}	$p_4^t=p_4+\sum\Delta p_{4j}$	$q_4^t=q_4+\sum\Delta q_{4j}$	$C_4^t=p_4^t \cdot q_4^t$	$C_4^t - C_4$	$C_4^t/C_4 - 1$
5	p_5	q_5	$C_5=p_5 \cdot q_5$	Δp_{51}	Δq_{51}	Δp_{52}	Δq_{52}	Δp_{53}	Δq_{53}	$p_5^t=p_5+\sum\Delta p_{5j}$	$q_5^t=q_5+\sum\Delta q_{5j}$	$C_5^t=p_5^t \cdot q_5^t$	$C_5^t - C_5$	$C_5^t/C_5 - 1$
Total			$C = \sum C_i$									$C^t = \sum C_i^t$	$C^t - C$	$C^t/C - 1$

Table 2. Numerical example without impact uncertainty.

	Anchor Budget at time t = 0			Event Impact Matrix, analysed at time t						Risk Budget at time t				
				1		2		3						
Activity	**p**	**q**	**C**	**Δp**	**Δq**	**Δp**	**Δq**	**Δp**	**Δq**	**p**	**q**	**C^t**	**ΔCost**	**ΔCost %**
1	100	1.000	100.000	20	100		25			120	1.125	135.000	35.000	35,0%
2	50	10.000	500.000						–200	50	9.800	490.000	–10.000	–2,0%
3	200	500	100.000	30						230	500	115.000	15.000	15,0%
4	1.000	150	150.000				10			1.000	160	160.000	10.000	6,7%
5								150	300	150	300	45.000	45.000	
Total			850.000									945.000	95.000	11,2%

Table 3. Event Impact Matrix using triangular uncertainty representation [a; c; b].

	Event Impact Matrix, analysed at time t					
	1		2		3	
Activity	**Δp**	**Δq**	**Δp**	**Δq**	**Δp**	**Δq**
1	[18; 20; 25]	[95; 100; 125]		[21; 25; 31]		
2					[–210; –200; –175]	
3	[25; 30; 45]					
4				[7; 10; 16]		
5					[145; 150; 165]	[280; 300; 350]
Total						

Table 4. Triple estimate Risk Budget (Anchor Budget of Table 2 and Event Impact Matrix of Table 3).

	Risk Budget at time t				
Activity	**p**	**q**	**C^t**	**ΔCost**	**ΔCost %**
1	[118; 120; 125]	[1.116; 1.125; 1.156]	[131.688; 135.000; 144.500]	[31.688; 35.000; 44.500]	[31,7%; 35,0%; 44,5%]
2	50	[9.790; 9.800; 9.825]	[490.500; 490.000; 491.250]	[–10.500; –10.000; –8.750]	[–2,1%; –2,0%; –1,8%]
3	[225; 230; 245]	500	[112.500; 115.000; 122.500]	[12.500; 15.000: 22.500]	[12,5%; 15,0%; 22,5%]
4	1.000	[157; 160; 166]	[157.000; 160.000; 166.000]	[7.000; 10.000; 16.000]	[4,7%; 6,7%; 10,7%]
5	[145; 150; 165]	[280; 300; 350]	[40.600; 45.000; 57.750]	[40.600; 45.000; 57.750]	
Total			[931.288; 945.000; 982.000]	[81.288; 95.000; 132.000]	[9.6%; 11,2%; 15,5%]

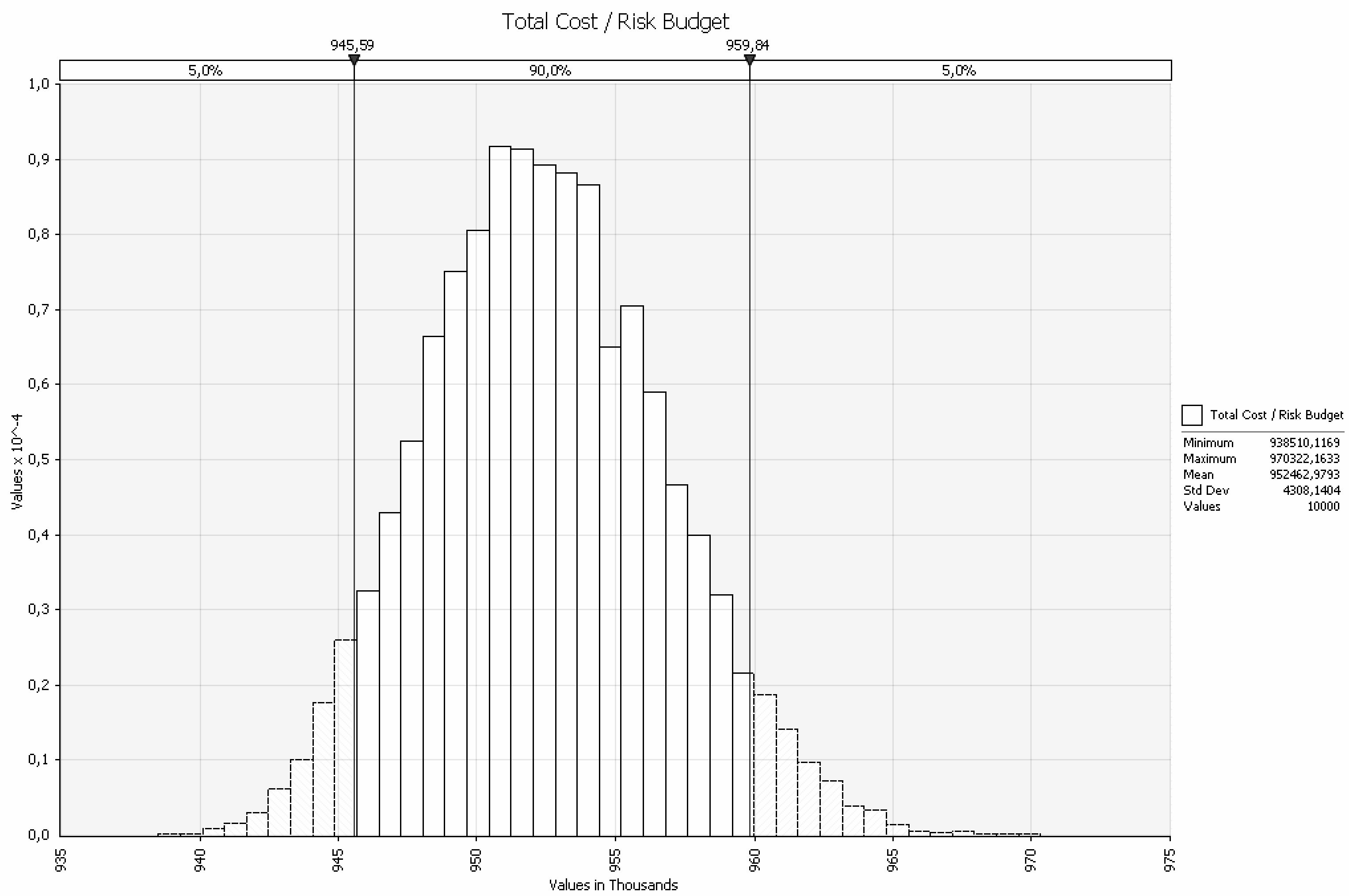

Figure 1. Triangular probability Monte Carlo simulation of Risk Budget. Uncorrelated input parameters. (Anchor Budget of Table 2 and Event Impact Matrix of Table 3).

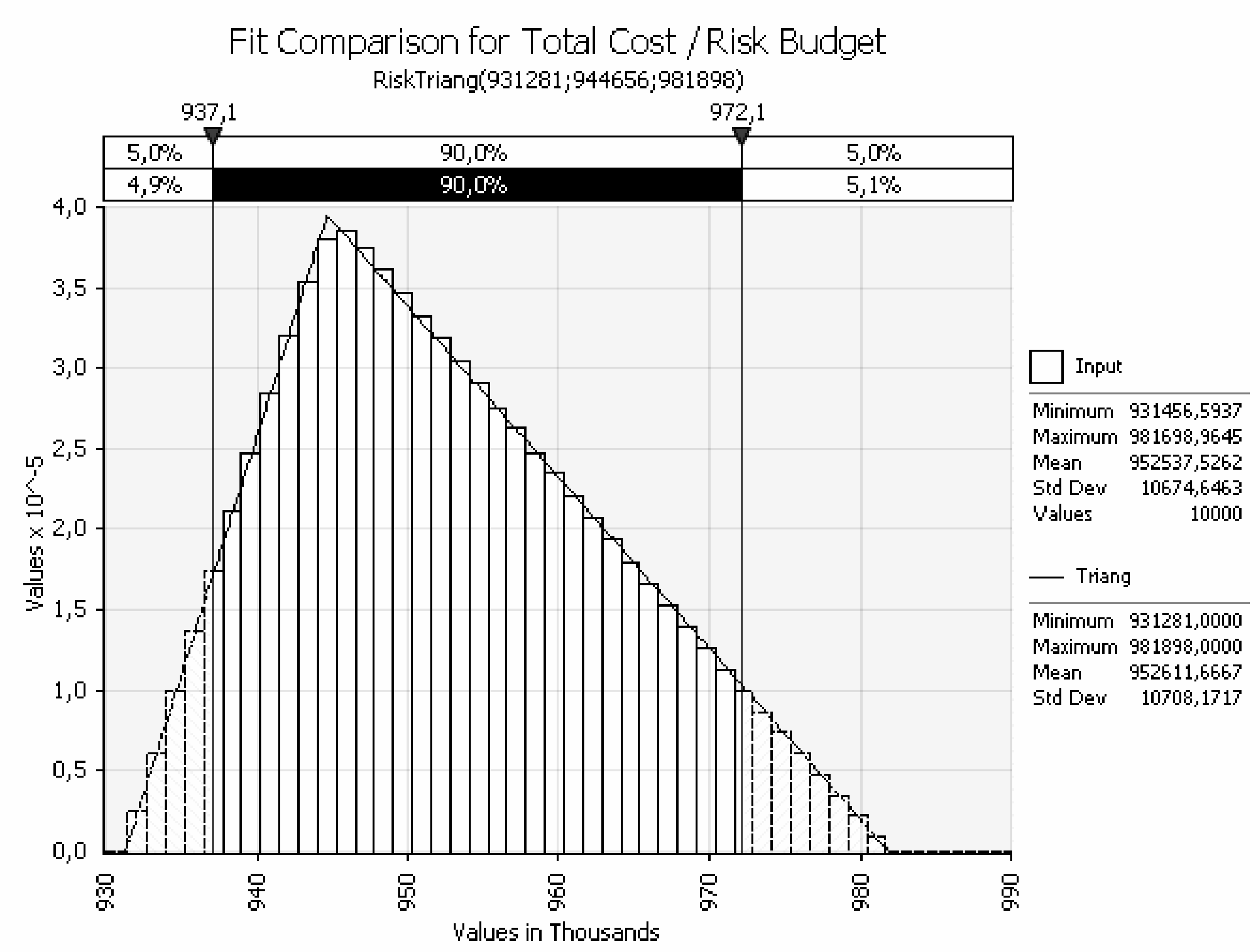

Figure 2. Triangular probability Monte Carlo simulation of Risk Budget. Fully correlated input parameters. (Anchor Budget of Table 2 and Event Impact Matrix of Table 3).